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A GENERAL FATIGUE PREDICTION METHOD BASED ON NEUBER NOTCH STRESSES AND STRAINS

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FOREWORD

This report is the result of an in-house effort under project 1347, "Structural Testing of Flight Vehicles", Task 134703, "Structural Testing Criteria". The manuscript was released by the author in October 1970 for publication as a technical report.

This technical report has been reviewed and is approved.

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ABSTRACT

A new combination of the Neuber parameter and stress-strain data is proposed and investigated for a completely general graphic analysis of cycle-by-cycle notch stress level. The proposed analysis is applied to two common aircraft structural materials, 2024-T4 and 7075-T6. Life to failure predictions based on the graphically derived notch stress levels compare very favorably with constant stress amplitude notched coupon results.

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SYMBOLS

E	Modulus of Elasticity
S	Nominal Net Section Stress
e	Nominal Net Section Strain
σ	Notch Root Stress Level or Unnotched Coupon Stress Level
ε	Notch Root Strain Level or Unnotched Coupon Strain Level
$\Delta S, \Delta e$, $\Delta\sigma, \Delta\varepsilon$	Change in Nominal or Point Stress or Strain Levels from Reversal to Reversal
ε_p	Plastic Component of Strain
ε_e	Elastic Component of Strain
K_T	Theoretical Elastic Stress Concentration Factor
K_σ	Actual Stress Concentration Factor, $\frac{\sigma}{S}$
K_ε	Actual Strain Concentration Factor, $\frac{\varepsilon}{e}$
n	Strain Hardening Exponent, Slope of True Stress - True Plastic Strain Curve
K	Strength Coefficient, σ at $\varepsilon_p = 1.$ on True Stress Plastic Strain Curve
$\Sigma\frac{n}{N}$	Ratio of Actual Cycles to Failure at Prescribed Loading Condition to the Predicted Cycles to Failure
R	Ratio of S_{min}/S_{max}

SECTION I

INTRODUCTION

A rational, inexpensive, and successful cumulative damage prediction has been searched for by stress analysts and designers for many years. Contemporary elastic Miner type analyses result in $\Sigma \frac{n}{N}$ values from 0.25 to 9.9 (Reference 1). Among the best of the fatigue life prediction methods are those that attempt to determine plasticity induced residual stress levels at notches and other discontinuities.

Smith (Reference 2) proposed a residual stress model based on an assumption that notch strain was proportional to nominal load. Notch stress was then determined from uniaxial monatonic stress-strain behavior. Smith notes and uses the experimental observation of elastic decrease in stress-strain behavior in axial loaded specimens after tensile yield. The final prediction method, however, requires a full scale structural article tested to failure at less than 10^4 cycles of R greater than zero. This approach does not readily lend itself to preliminary fatigue design predictions, but should assist in planning "fixes" for existing structure.

Impellizzeri (Reference 3) suggests an analysis based on notch residual stress assuming the first reversal notch stress level is determinable from the material's stress-strain behavior as governed by the Neuber (Reference 4) parameter $K_T = (K_\sigma K_\varepsilon)^{1/2}$. The second and subsequent reversal analysis is accomplished in like manner assuming

the uniaxial compression behavior is applicable. As in the previously discussed analysis, Impellizzeri assumes notch strain to be proportional to nominal stress. The analysis suggested is tedious and not readily adaptable to preliminary design prediction since the method requires a detailed constant amplitude S-N curve for identical or very similar structures before a spectrum load prediction can be made. Instead of the required extensive constant amplitude testing of candidate structures before making a prediction, it would seem more economical to test actual structures to the prospective load spectrum. If the data required by Impellizzeri are available however, extremely accurate life predictions are obtainable.

Wetzel, Morrow and Topper (Reference 5) suggest an experimental program of a series of unnotched coupons, spectrum stress-strain controlled according to the Neuber parameter, in order to obtain a prediction of fatigue life to cracking of a prospective structure. Wetzel (Reference 6) advises that residual stress levels can be obtained from the first few cycles of constant amplitude Neuber-controlled specimens. The experimenter can then consult an S-N curve for unnotched material at that residual stress level and obtain a life prediction without continuing the test. Either technique requires much experimental data to allow predictions to be made, but testing of simple coupons is many orders of magnitude less expensive than full-scale testing of one or a few full-scale components.

This paper suggests and investigates the combination of certain

basic experimental and analytical aspects of these above techniques into a generally applicable curve for use in the calculation of residual stress levels, thus increasing the accuracy and reliability of fatigue life predictions.

SECTION II

DERIVATION OF A GENERAL CYCLIC NOTCH STRESS VS NOMINAL STRESS RELATION

Any analysis requires several basic guidelines or rules to follow. Among the rules utilized in this derivation are:

1. Reversal-by-reversal* notch stress-strain behavior follows the analysis suggested by Neuber (Reference 4). That is, notch stress and strain are related to the nominal stress level by the equation $\Delta\sigma\Delta\varepsilon = \frac{(K_T\Delta S)^2}{E}$. This particular relation assumes gross yielding does not occur in the net section.
2. Upon release of load after yielding in tension or compression, the notch material acts as if it were elastic until yielding occurs in the opposite direction.
3. The stress-strain behavior in compression is assumed identical to that in tension unless experimental data prove otherwise.

Figures 1, 2 and 3 show tentative $K_T\Delta S_1$ vs σ curves of first and second reversal for 7075T6 aluminum alloy. The horizontal axis is equal to the elastic stress-concentration factor multiplied by the net nominal stress level for the first reversal. The vertical axis is the actual notched specimen stress level at the discontinuity according to the Neuber analysis. The first reversal curves were plotted from the

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*A reversal is general nomenclature designating that period between two relative peaks. Reversals are so named since the direction of loading is reversed following each relative peak. Two reversals constitute a cycle of load application.

Neuber parameter $K_T\Delta S = (\Delta\sigma\Delta\varepsilon E)^{1/2}$ relating the change in notch stress and strain to the change in nominal stress. The assumption made was that on the first reversal the notch material would react as a monotonic uniaxial tensile member. Monotonic stress and strain are describable by the strain hardening relation $\sigma = K\varepsilon_p{}^n$. The total strain in the specimen is a combination of the elastic and plastic contribution.

$$\varepsilon = \varepsilon_e + \varepsilon_p$$

In terms of stress the strain term becomes: $\varepsilon = \sigma/E + (\sigma/K)^{1/n}$

Therefore for the first reversal curve $K_T\Delta S_1$ and σ are related by:

$$K_T\Delta S_1 = \{E\Delta\sigma[\Delta\sigma/E+(\Delta\sigma/K)^{1/n}]\}^{1/2}$$

Figure 1 shows Notch Stress Level versus $K_T\Delta S_1$ derived from the above equation for 7075T6 Aluminum Alloy. The strain hardening values "K" and "n" were those reported in Reference 5. Subsequent decrease in nominal load on the specimen results in elastic notch stress decrease equal to $K_T\Delta S_2$. A locus of constant $K_T\Delta S_2$ can be incorporated into the $K_T\Delta S_1$ - σ curve as a curve parallel but vertically displaced from the first reversal notch stress. A family of such parallel but displaced lines can be drawn to obtain a curve giving notch stress level at the first and second reversals for any general geometry and stress level. The notch stress level for the first reversal is found on the first reversal curve at an ordinate value of $K_T\Delta S_1$. The second reversal notch stress level is found by dropping vertically from the first reversal position to the second reversal curve of the proper value. Figure 2 shows the first reversal curve and a family of

elastic second reversal curves. Points A and B are sample notch stress levels for the first and second reversals respectively.

Compression yielding limits the minimum notch stress level and alters the slope of the second reversal curves by flattening them out somewhat. As compression yield would occur, the notch stress per change in $K_T\Delta S_2$ would decrease. The first area to see change would be at low $K_T\Delta S_1$ values since the elastic notch stress is lowest for equal $K_T\Delta S_2$. This portion of the lines would tend to flatten before that portion at higher $K_T\Delta S_1$. Without experimental data to establish the total general curves, it is sufficient to assume that the stress-strain behavior in compression is the negative image of that in tension. In that case the negative absissa of the general $K_T\Delta S_2$ curves would be the negative of the first reversal notch stress values at corresponding $K_T\Delta S_1$. Figure 3 shows the compression yield affected general curve for this 7075T6 material.

Figures 4 and 5 show general $K_T\Delta S_1$ vs σ curves for 7075T6 and 2024T4 Aluminum Alloys respectively. The lines of constant $K_T\Delta S_2$ in the case of 2024T4 after apparent compression yielding were determined from the locus of minimum stress values reported by Crews (Reference 7,8) at R=0 and R=-1. The data plotted include K_T values of 2, 4, and 6, demonstrating general applicability of the curve. Table 1 shows the monotonic stress-strain parameters used for the two materials investigated.

Figure 6 shows a $K_T\Delta S_1$ vs ε for 2024T4 derived from the Neuber

relationship applied to Figure 5. The compression yield affected portion of Figure 5 was determined from the Crews' notch stress level data. The strain level data should follow closely the calculated strain levels if the Neuber parameter is applicable. The Crews' strain data for $K_T = 2$, 4 and 6 are seen to fall near and bracket the first reversal curve, the $R = 0$, and $R = -1$ notch strain level predictions. The fact that the $R = 0$ minimum strain level is greater than zero for $K_T \Delta S_1$ over 40 Ksi indicates that the notch strain level is not proportional to the nominal stress.

SECTION III

ANALYSIS TECHNIQUE

The proposition is that the designer can use uniaxial material behavior data and some experimental data to develop a rational notched specimen cumulative damage prediction. The prediction is based upon a plot of notch stress level vs $K_T \Delta S_1$ used in conjunction with an extremely simple parallel line overlay to predict residual stress levels in notched coupons subjected to cyclic loading.

The calculation of residual stress level and fatigue life in constant amplitude loading is done as follows:

1. Determine notched specimen $K_T \Delta S_1$; find σ on "first reversal curve" of $K_T \Delta S_1$ vs σ curve. This is the σ_{max} value.

2. Determine $K_T \Delta S_2$ value from constant amplitude nominal stress history. Go to $K_T \Delta S_1$ vs σ curve at $K_T \Delta S_1$, σ_{max} position. Drop vertically until the required $K_T \Delta S_2$ is reached. This is the σ_{min}.

3. These values of σ_{max} and σ_{min} are the notch stress values during the constant amplitude loading of notched coupons. Fatigue life of the notched coupon is then estimated by determining the fatigue life for unnotched coupons between σ_{max} and σ_{min} constant amplitude.

Thus, knowing the geometry of the specimen (i.e. K_T) and the testing conditions (R, S_{max}, S_{mean}, etc.), a fatigue life prediction can be made based on the residual stress level at the notch found in the $K_T \Delta S_1$ - σ curve.

For example, assume 2024T4 material notched to $K_T = 2.0$ and tested at $R = 0.0$ with $S_{max} = 30,000$ psi. From Figure 5 at $K_T \Delta S_1 = 60,000$, σ_{max} is found to be 50 ksi. Dropping vertically by $K_T \Delta S_2 = -60,000$ gives $\sigma_{min} = -10$ ksi.

In the example the notch stress level during cycling could be accurately described as -10 ksi to +50 ksi giving $\sigma_{alt} = \pm 30$ ksi and $\sigma_{mean} = +20$ ksi. Fatigue life prediction in constant ΔS amplitude loading is then determined from the Modified Goodman Diagram of unnotched axial loaded specimens of the desired material. Unnotched S-N data (Reference 9) indicate a life of ≈ 70,000 cycles for this condition.

Spectrum loading residual stress level prediction is somewhat more involved especially if significant plasticity occurs, but can be handled using a parallel line overlay essentially like that used for $K_T \Delta S_2 = C$ curves. The overlay can be "slid" vertically and $K_T \Delta S$ values introduced reversal by reversal to obtain cyclic unnotched specimen stress levels. If the overlay would indicate that the stress level exceeds the first reversal curve, implying yielding occurred, for instance, one would slide the horizontal reference level to operate on a different $K_T \Delta S_1$ position while allowing for the yield behavior. The main problem lies in the determination of the yield behavior in compression. At this stage in the development of this analysis, it was felt sufficient to assume, unless otherwise known, that the material behavior in compression is equal to that in tension.

SECTION IV

RESULTS AND DISCUSSION

Figures 7, 8, 9 and 10 show the results of actual notched specimen fatigue tests and the prediction based on the described method. The notched specimen results are from a number of different investigators (References 6, 10, 11, 12) who tested specimens with K_T of 2.0, 2.4, 2.5 and 4.0. The data exhibited minor variations in heat treatment. It was assumed that the fatigue properties were identical. The prediction shows very good correlation with the data in all cases. Elimination of the scatter in the test results shows the prediction to be almost uniformly conservative. This would be expected because some crack propagation must occur after the fatigue crack has been initiated. The data for R = -1.0 in Figures 8 and 10 have been omitted since they are the same as those plotted for $K_T S_{mean} = 0$ ksi in Figures 7 and 9 respectively. No data on notched coupons at R = 0.5 was found but the predicted fatigue life curve was plotted to indicate the general trend.

The prediction analysis used in this paper assumed that the first reversal of the constant amplitude loading was of positive sign and of the proper amplitude. Actual startup problems of the specimen could have a drastic effect on the fatigue life. If, for example, the first or any subsequent load application had a value higher than that expected, a more compressive residual stress load than normal could result. The fatigue life of this component would be increased because

of the lowering of the effective mean stress. In spectrum loading situations a series of small amplitude cycles following a compressive load peak that caused compression yielding (and therefore a tensile residual stress) would be much more damaging than the same set following a tensile load peak although the elastic analysis technique would give equal weight in both situations. Other ambiguities exist since a common closed loop system startup technique consists of "nudging" the mean stress and stress amplitude controls over a span of several cycles until the desired stress level is reached. The actual notch stress level would be difficult to predict in the case that compressive yielding were possible. From these intuitive examples it is seen that simple permutations of the startup technique could add to (or explain) the apparent scatter seen in fatigue results. Carrying this argument further, seemingly minor variations in early life aircraft load experience can possibly explain the large scatter observed in aircraft fatigue lives. The general fatigue analysis approach given here could be of use in giving quantitative answers to the problems posed. The importance of the notch residual stress level induced in the first few loading cycles should not be overlooked.

SECTION V

CONCLUSIONS

In most present linear analysis techniques it appears that unnotched coupon data cannot be accurately applied to the fatigue life prediction of notched coupons which, in turn, cannot be used to predict life of built up structures. Therefore, in the qualification of a new material and/or structural design several steps are required, up to and including much expensive testing of full-size components, to obtain confidence to build a fleet of craft utilizing the new material or design. Even in much used, contemporary materials the best design practices sometimes produce expensive mistakes.

This analysis form shows much promise as a means of accurately predicting notched coupon and, perhaps, even total structure fatigue life from simple unnotched coupon S-N data by calculating of cycle-by-cycle residual stress levels. The simplicity and generality of the technique are the best points in favor of its credibility and use-ability. At this stage large cyclic plasticity does not appear readily handleable as the Neuber analysis itself is limited to cases where extreme net section yielding does not occur.

SECTION VI

RECOMMENDATIONS FOR FURTHER WORK

1. Experimentally determine the general $K_T \Delta S_1 - \sigma$ curve for each material of engineering interest. This can be done with Neuber controlled smooth specimens. A better definition of the effect of compression yield on the notch stress level prediction is necessary for accurate fatigue damage prediction.

2. Automate the procedure to facilitate fatigue prediction of complicated spectrum loaded structures.

3. Study effect of cyclic stress relaxation on the analysis.

SECTION VII

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TABLE 1

MONATONIC STRESS STRAIN RELATIONS

Designation	2024T4*	7075T6 [5]
Modulus of Elasticity E,Ksi	10,300.	10,300.
0.2% Offset Yield Strength S_{TY},Ksi	58.	68.
Strength Coefficient K, Ksi	100.	120.
Strain Hardening Exponent n	.115	.113

*(This investigation)

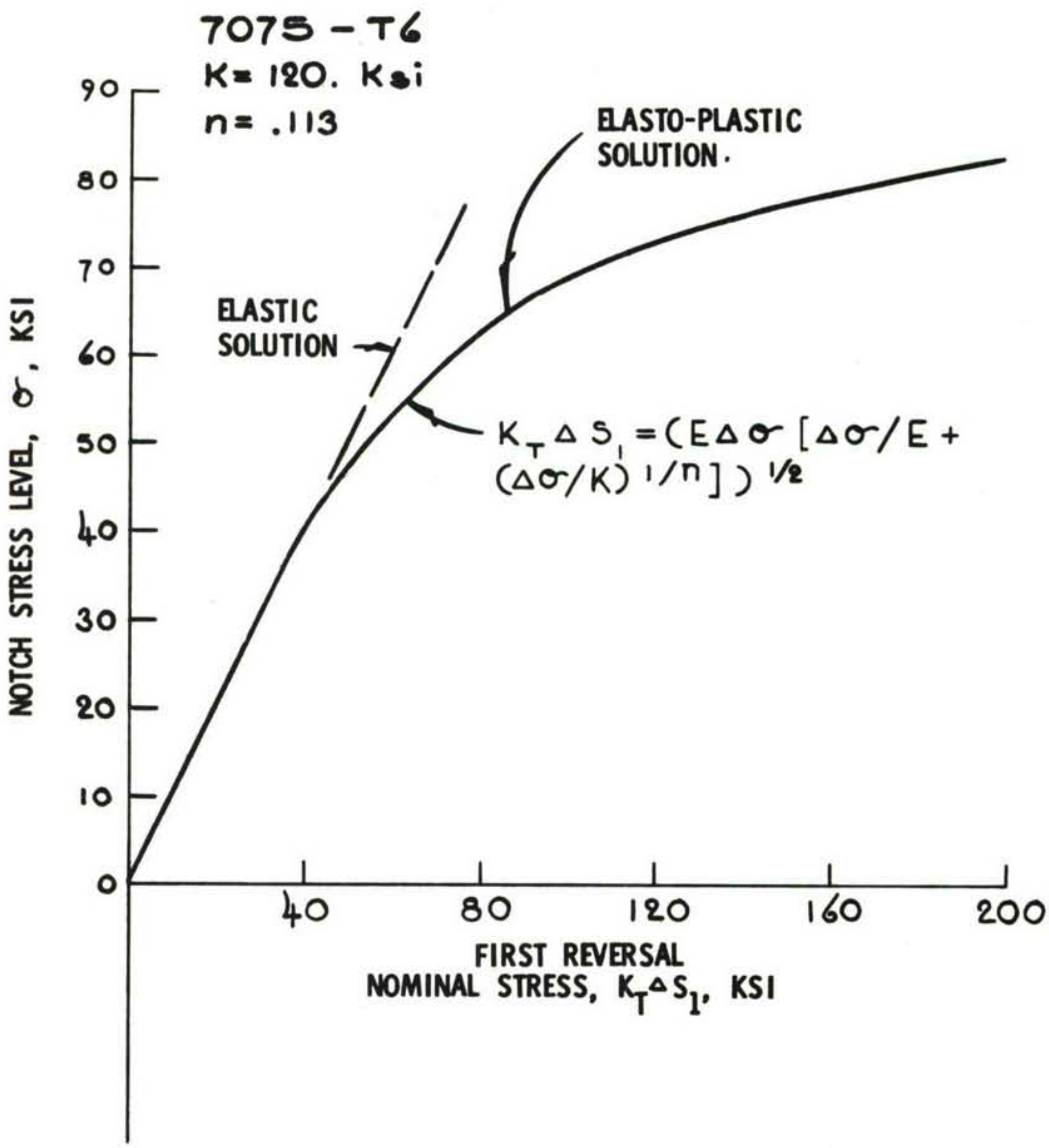

FIGURE 1. FIRST REVERSAL NOTCH STRESS VS PRODUCT OF NOMINAL STRESS AND STRESS CONCENTRATION FACTOR FOR 7075-T6 BASED ON NEUBER NOTCH PARAMETER, $K_T = (K_\sigma K_\epsilon)^{1/2}$

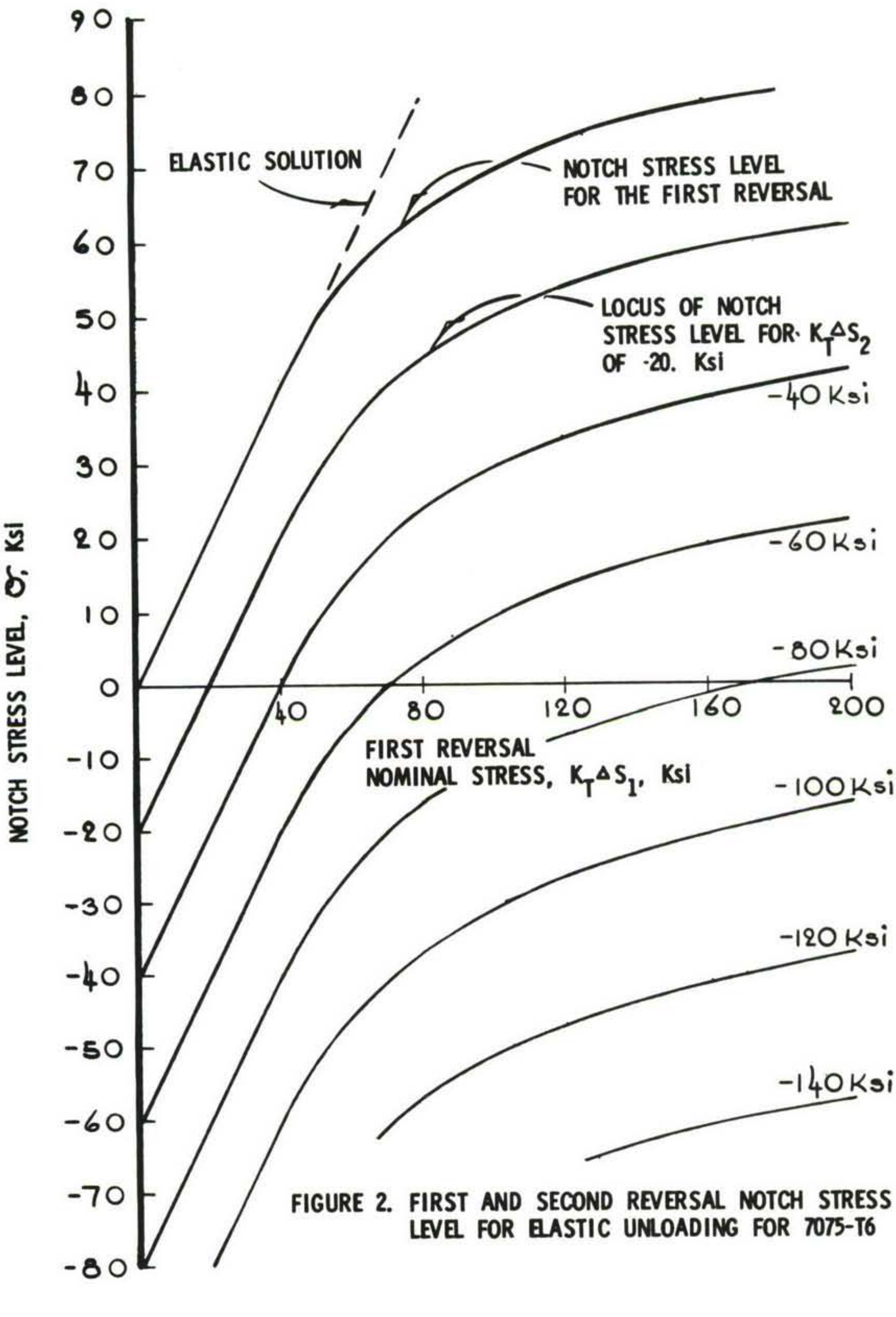

FIGURE 2. FIRST AND SECOND REVERSAL NOTCH STRESS LEVEL FOR ELASTIC UNLOADING FOR 7075-T6

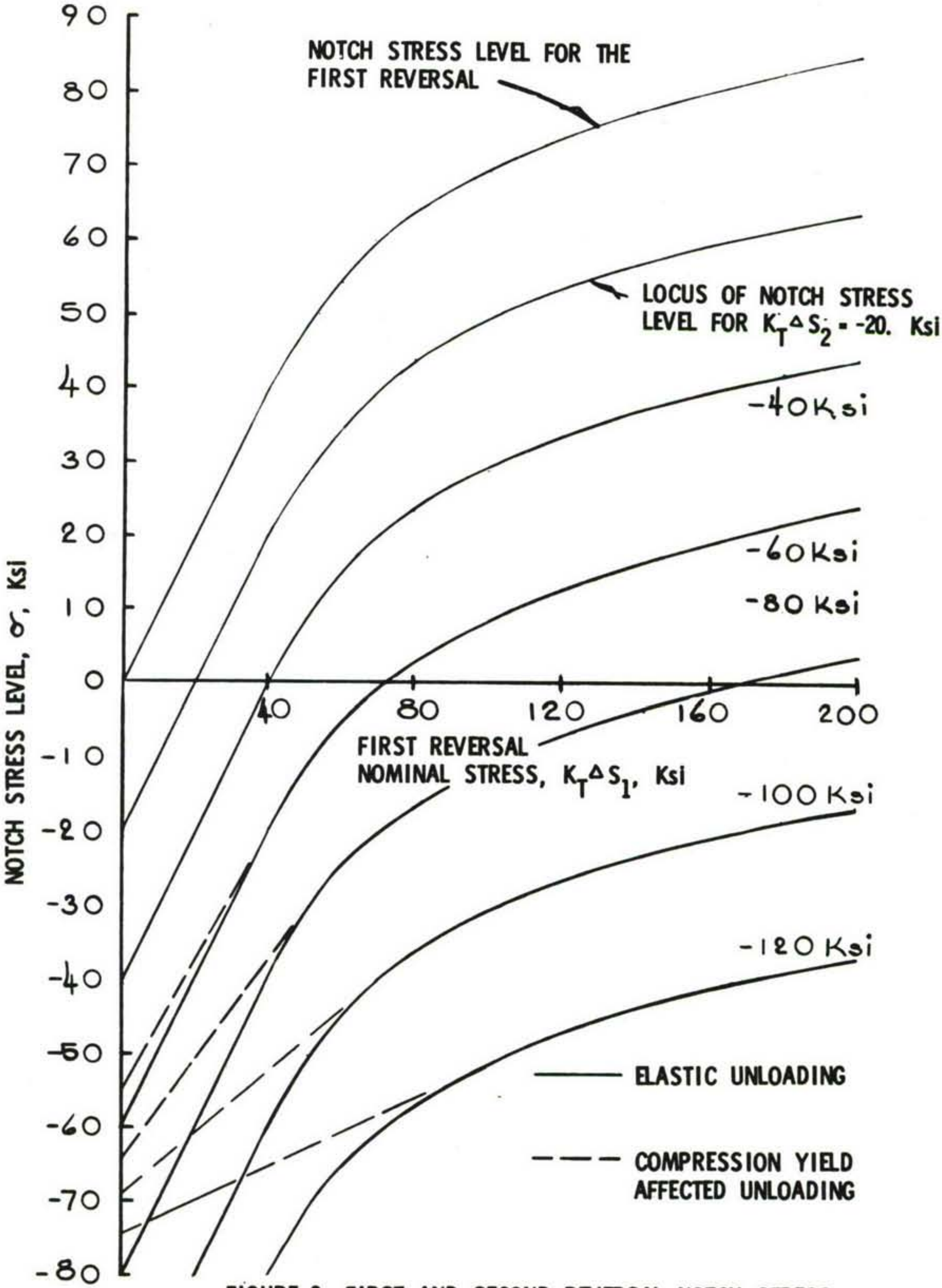

FIGURE 3. FIRST AND SECOND REVERSAL NOTCH STRESS LEVEL FOR ELASTIC-PLASTIC UNLOADING FOR 7075-T6

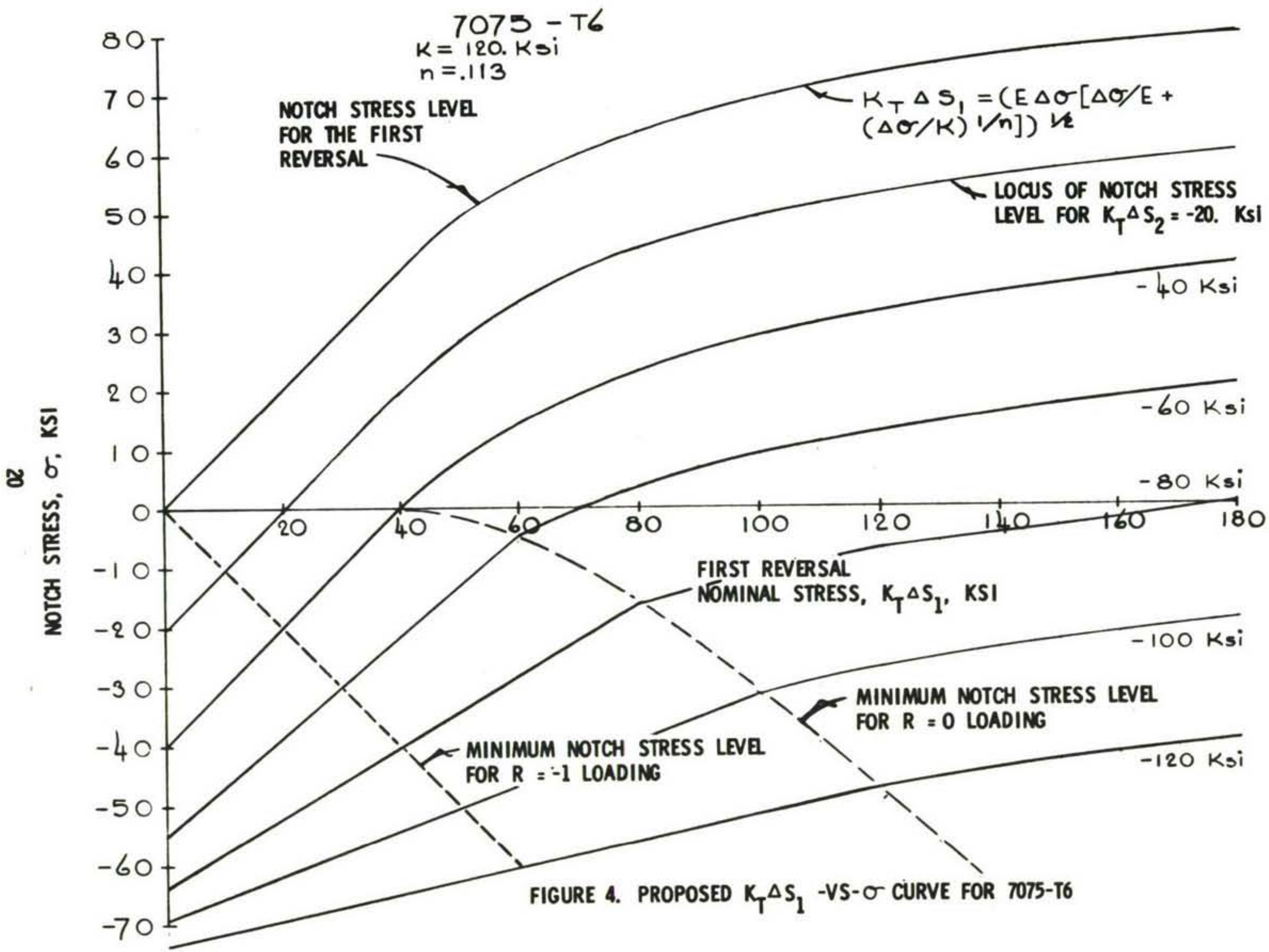

FIGURE 4. PROPOSED $K_T \Delta S_1$ -VS-σ CURVE FOR 7075-T6

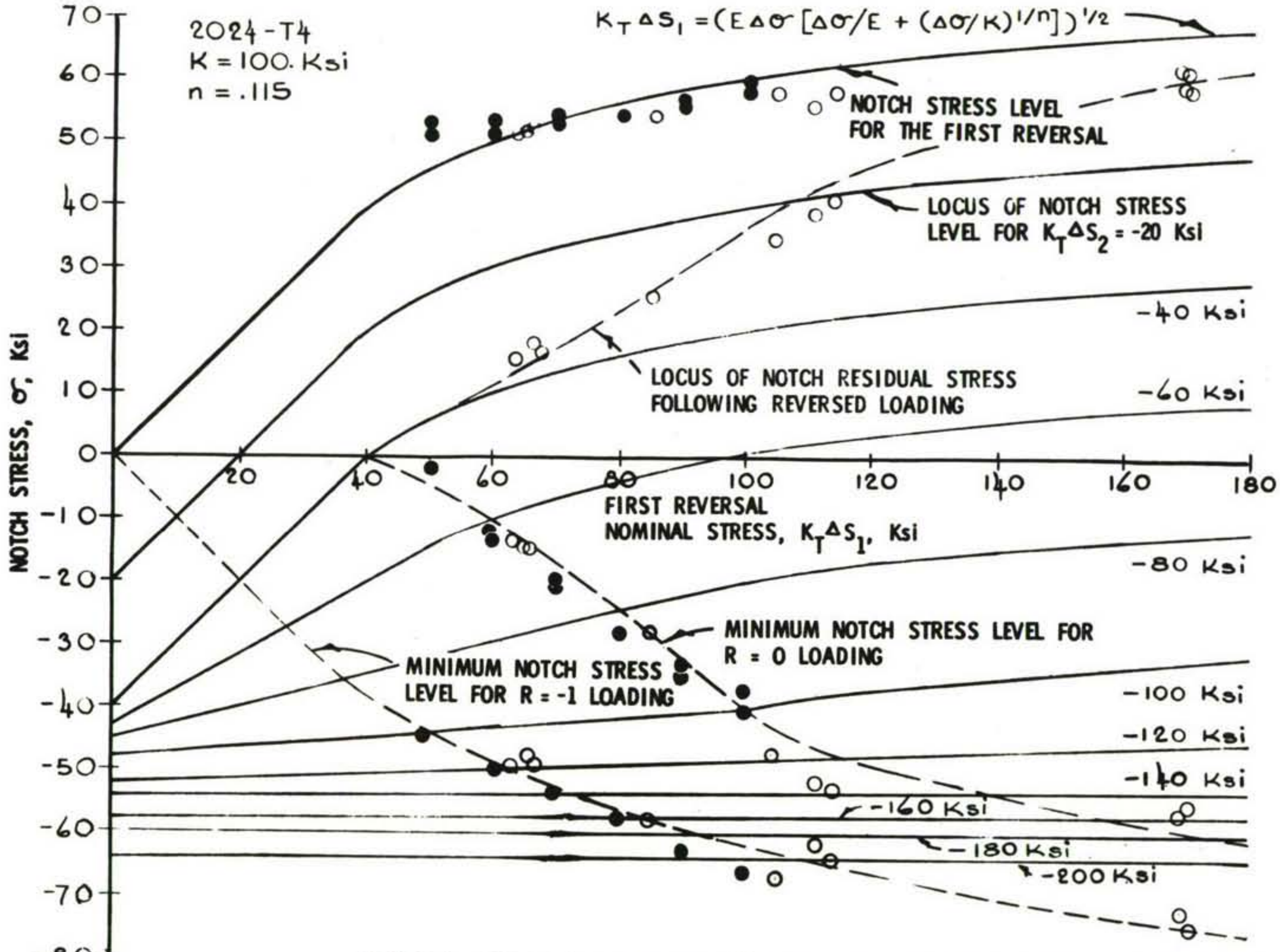

FIGURE 5. PROPOSED $K_T \Delta S$-VS-σ CURVE FOR 2024-T4 WITH COMPARISON OF MEASURED NOTCH STRESS LEVELS

● MEASURED NOTCH STRESS(7)

○ MEASURED NOTCH STRESS(8)

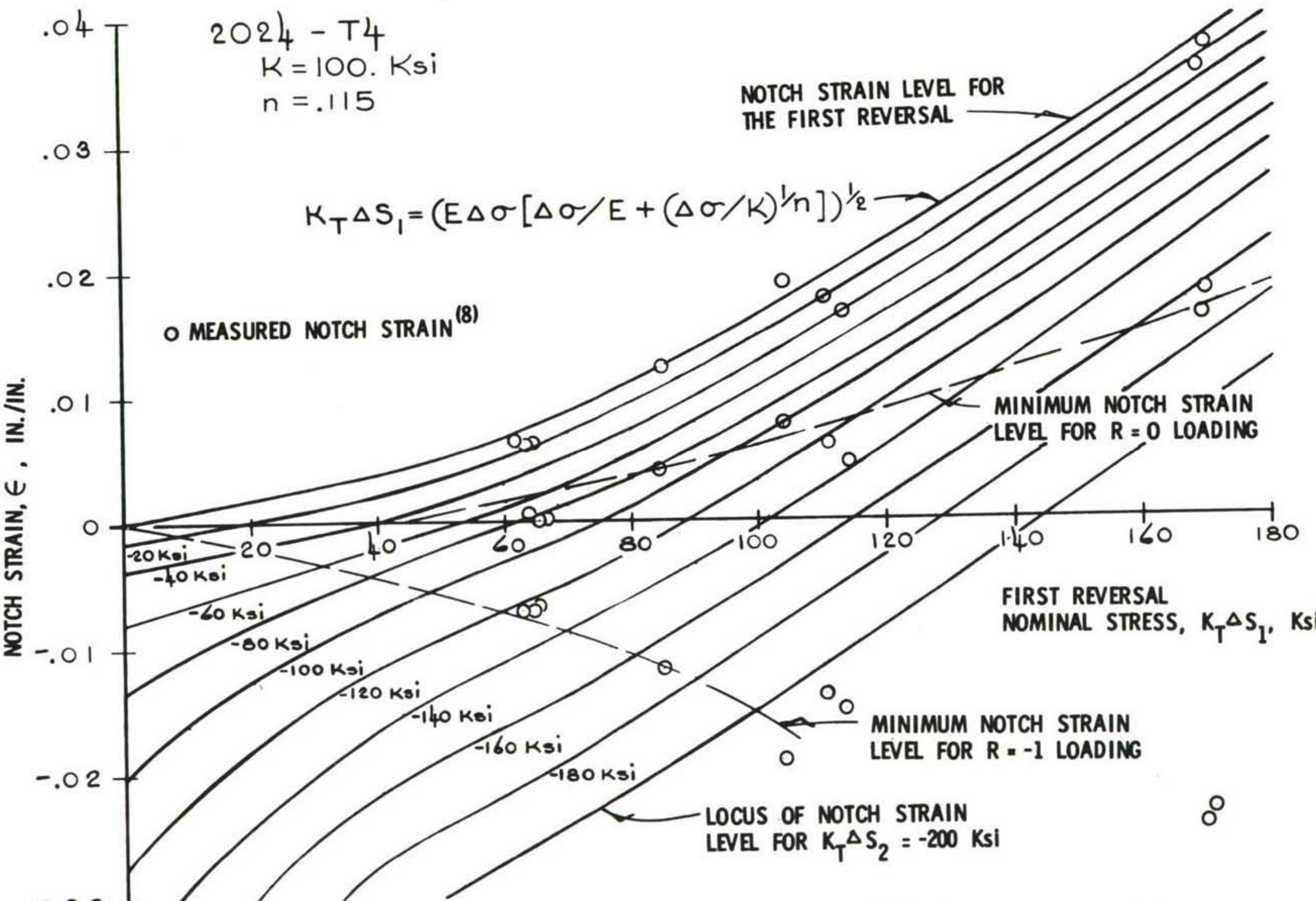

FIGURE 6. PROPOSED $K_T \Delta S_1$ - VS - ∈ CURVE FOR 2024-T4 WITH COMPARISON OF MEASURED NOTCH STRAIN LEVELS

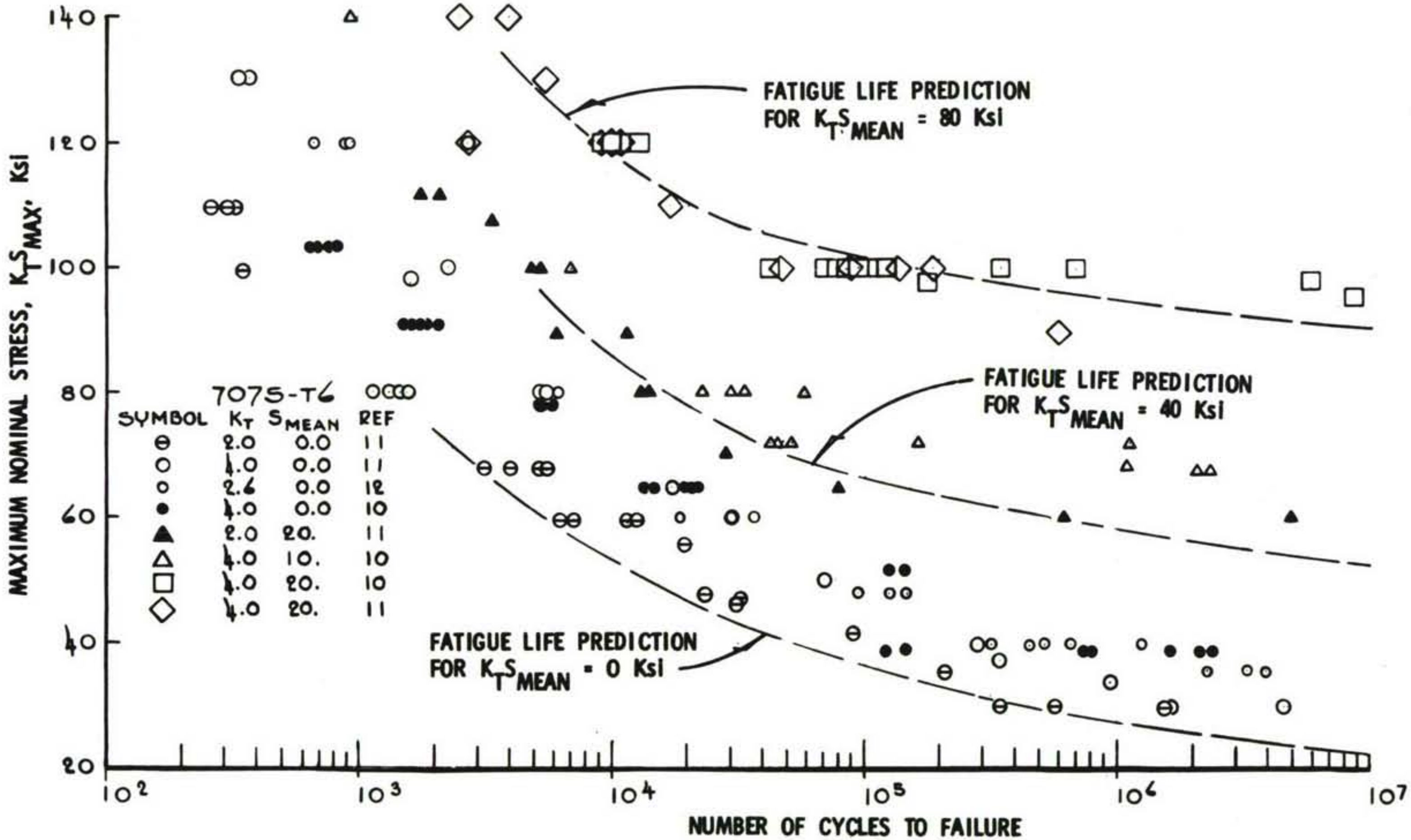

FIGURE 7. COMPARISON OF FATIGUE DATA AND PREDICTION TECHNIQUE FOR 7075-T6 AT SEVERAL MEAN STRESSES

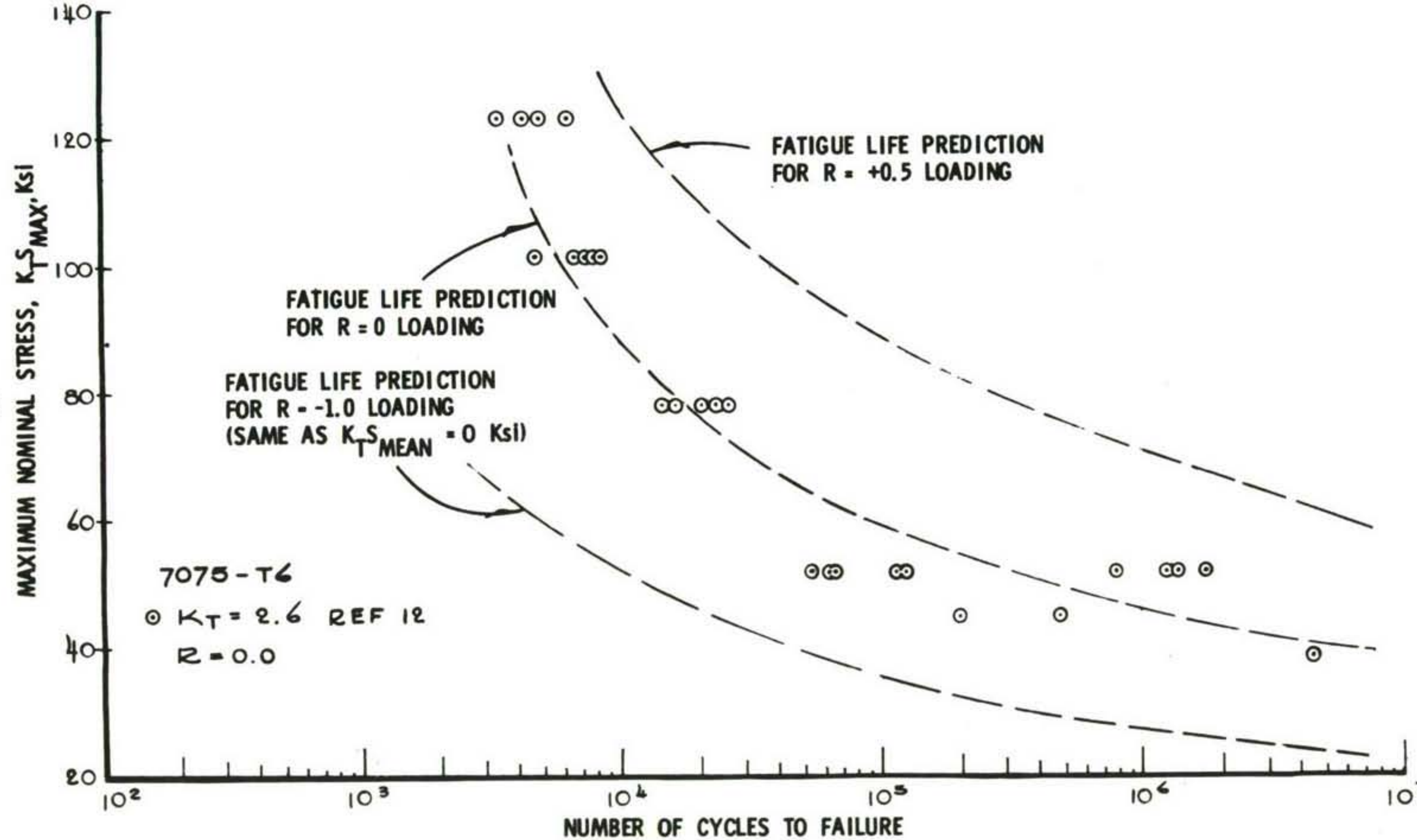

FIGURE 8. COMPARISON OF FATIGUE DATA AND PREDICTION TECHNIQUE FOR 7075-T6 AT SEVERAL STRESS RATIOS

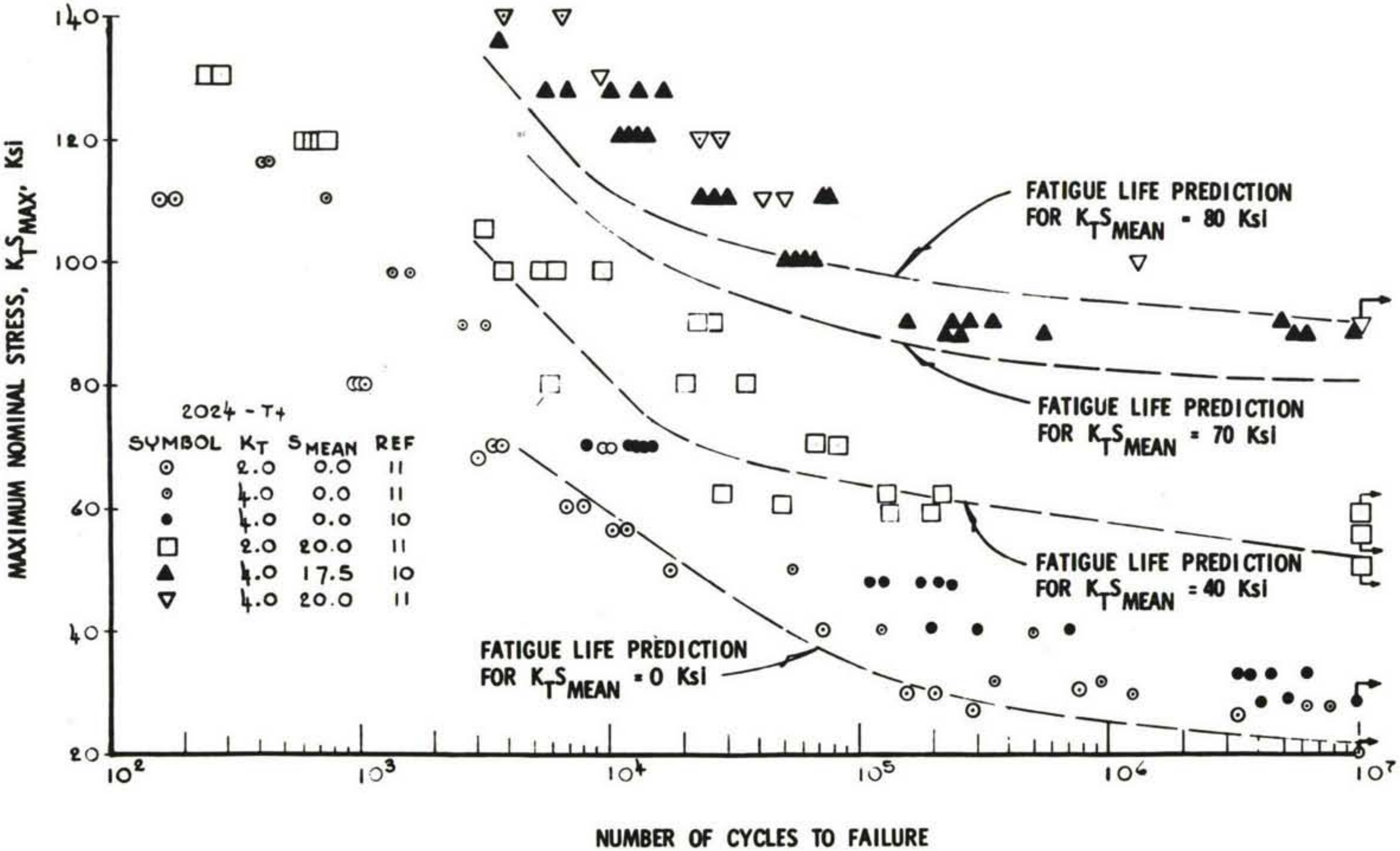

FIGURE 9. COMPARISON OF FATIGUE DATA AND PREDICTION TECHNIQUE FOR 2024-T4 AT SEVERAL MEAN STRESSES

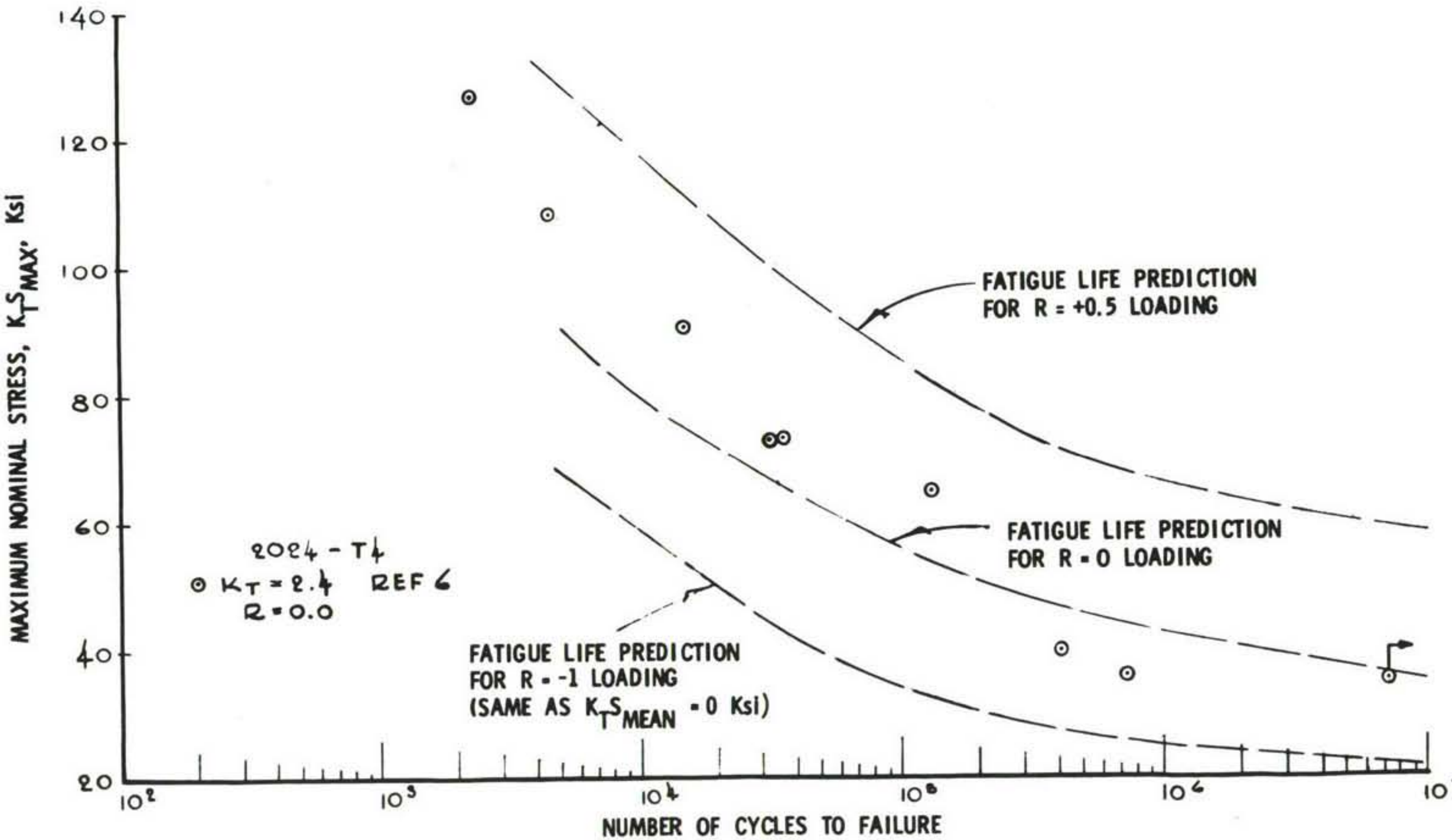

FIGURE 10. COMPARISON OF FATIGUE DATA AND PREDICTION TECHNIQUE FOR 2024-T4 AT SEVERAL STRESS RATIOS

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13. ABSTRACT

A new combination of the Neuber parameter and stress-strain data is proposed and investigated for a completely general graphic analysis of cycle-by-cycle notch stress level. The proposed analysis is applied to two common aircraft structural materials, 2024-T4 and 7075-T6. Life to failure predictions based on the graphically derived notch stress levels compare very favorably with constant stress amplitude notched coupon results.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Fatigue Analysis						
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Solid Mechanics						